

8.2 The Quadratic Formula

$$ax^2 + bx + c = 0$$

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \boxed{\frac{b^2}{4a^2}} = -\frac{c}{a} + \boxed{\frac{b^2}{4a^2}}$$

$$\left(\frac{1}{2} \cdot \frac{b}{a}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)\left(x + \frac{b}{2a}\right) = \left(x + \frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{4a \cdot c}{4a^2} + \frac{b^2}{4a^2} \quad 4a^2 \text{ LCD}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\frac{x + \frac{b}{2a}}{\frac{2a}{2a}} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \rightarrow \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{quadratic formula}$$

$$3x^2 + 5x - 7 = 0$$
$$a = 3 \quad b = 5 \quad c = -7$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Traditional quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-7)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{25 + 84}}{6}$$

$$x = \frac{-5 \pm \sqrt{109}}{6}$$

$$2x^2 - 4x + 7 = 0$$

$$a = 2 \quad b = -4 \quad c = 7$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(7)}}{2(2)}$$

$$-40$$
$$2 \cdot 20$$
$$4 \cdot 10$$

$$x = \frac{4 \pm \sqrt{16 - 56}}{4}$$

$$x = \frac{4 \pm \sqrt{-40}}{4}$$

$$x = \frac{4 \pm 2i\sqrt{10}}{4}$$

$$x = \frac{2 \pm i\sqrt{10}}{2}$$

$$x = \frac{2}{2} \pm \frac{\sqrt{10}}{2} i \rightarrow 1 \pm \frac{\sqrt{10}}{2} i$$

The Brazilian Method of Quadratic Formula

$$2x^2 - 4x + 7 = 0$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$D = b^2 - 4ac$$

discriminant

- 1) find D
- 2) find \sqrt{D}
- 3) put together and simplify.

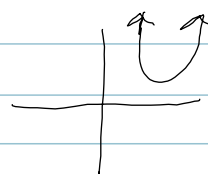
$D > 0$ 2 real solutions



$D = 0$ 1 real solution



$D < 0$ 2 imaginary solutions



$$\begin{aligned} 1) \quad D &= b^2 - 4ac \\ &= (-4)^2 - 4(2)(7) \\ &= 16 - 56 \\ &= -40 \end{aligned}$$

$$2) \quad \frac{\sqrt{-40}}{2i\sqrt{10}}$$

$$\begin{array}{r} 2 \overline{)40} \\ \underline{20} \\ 20 \\ \underline{20} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

$$3) \quad x = \frac{-b \pm \sqrt{D}}{2a}$$

$$x = \frac{-(-4) \pm 2i\sqrt{10}}{4}$$

$$x = \frac{4 \pm 2i\sqrt{10}}{4}$$

$$x = \frac{2 \pm i\sqrt{10}}{2}$$

Vertex formula

$$y = 3(x-2)^2 + 4$$

shape $\uparrow\uparrow$

$$(h, k) (2, 4)$$

$$a = 3 + \text{up}$$

is stretched

(h, k) version of the
quadratic formula

$$x = h \pm \sqrt{\frac{-k}{a}}$$

$$(h, k) (2, 4)$$
$$a = 3$$

$$x = 2 \pm \sqrt{\frac{-4}{3}}$$

$$x = 2 \pm \frac{2i}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = \frac{2 \pm 2i\sqrt{3}}{3}$$

$$3x^2 - 6x - 7 = 0$$

$$a = 3 \quad b = -6 \quad c = -7$$

$$1) D = b^2 - 4ac$$
$$(-6)^2 - 4(3)(-7)$$
$$36 + 84$$
$$120$$

$$2) \sqrt{D} = \sqrt{120}$$
$$2\sqrt{30}$$

$$\begin{array}{r} 2 \overline{)120} \\ \underline{2 \overline{)60}} \\ 2 \overline{)30} \\ \underline{3 \overline{)15}} \\ 5 \end{array}$$

$$3) \frac{-b \pm \sqrt{D}}{2a} \rightarrow \frac{6 \pm 2\sqrt{30}}{6}$$

$$\frac{3 \pm \sqrt{30}}{3}$$